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1978 J. Phys. A: Math. Gen. 11 2083

(http://iopscience.iop.org/0305-4470/11/10/024)

J. Phys. A: Math. Gen., Vol. 11, No. 10, 1978. Printed in Great Britain

Quantisation around superparametrised finite-energy classical solutions

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Received 15 May 1978

Abstract. For a two-dimensional field theory of bosons and fermions, we construct, using Dirac's method, the quantum theory around finite-energy classical solutions with arbitrary bosonic and fermionic parameters. Quantisation around a space-translated and a supertranslated solution of a supersymmetric field theory is a special case of our treatment.

1. Introduction

Finite-energy classical solitonic solutions to field equations are of current interest (Jackiw 1977). Such solutions are known to exist, for example, in the φ^4 theory and the sine–Gordon theory in two-dimensional space–time. In constructing the quantum theory around such solutions, one pays special attention to the so called translational mode. This is a zero-energy solution of the eigenvalue equation of perturbations. This mode exists by virtue of space-translational invariance of the theory, and can easily be obtained by infinitesimally space translating the static solution itself.

In theories containing fermion fields, one often encounters fermion zero-energy solutions (Jackiw and Rebbi 1976, Jackiw 1977). The origin of these may be traced to some supersymmetry (Salam and Strathdee 1975) of the field equations (Rossi 1977), at least for the particular background solitonic solution. Actually finite supersymmetry transformations (supertranslations) have been used (Baaklini 1977a) to construct fermion field solutions from known solutions in the bosonic sector of the theory. Since the Hamiltonian commutes with supertranslations, a supertranslated solution has the same energy as the 'embedded' bosonic one. Hence, the fermion solutions obtained by supertranslation are zero-energy solutions.

The translational and the supertranslational modes are dealt with, in constructing the quantum theory, by expanding around space-translated and supertranslated classical solutions, treating the translational and the supertranslational parameters as quantum variables (collective coordinates) on an equal footing with the other quantum field variables in the theory. Dirac's generalised theory for dealing with constrained systems (Dirac 1964) has been applied to construct the quantum theory around space translated solutions (Tomboulis and Woo 1976). A similar procedure has been applied to construct the quantum theory around a supertranslated classical solution (Baaklini 1978a).

Our purpose in this paper is to consider the construction of the quantum theory around general parametrised finite-energy classical solutions which contain arbitrary bosonic and fermionic parameters. These parameters may, for example, be due to bosonic or fermionic symmetry transformations. The formalism we develop affords greater generality and simplicity than previous treatments (Baaklini 1978a).

In § 2, we give some preliminaries about the two-dimensional model considered.

In § 3, we expand around a superparametrised classical solution. We define the canonical variables and obtain the constraints and the Hamiltonian.

In § 4, we study the algebra of the constraints and follow Dirac's method for putting the constraints strongly equal to zero and for defining the correct bracket rules.

In § 5, we discuss the canonical quantum theory as well as the path integral formulation.

In § 6, we give some remarks about the extension of this work to more interesting physical systems.

2. Preliminaries

We shall consider the theory described by the following Lagrangian in two-dimensional space–time,

$$L=\int \mathrm{d}x\, [\tfrac{1}{2}(\partial_\mu \Phi)^2+\tfrac{1}{2}\mathrm{i}\bar{\Psi}\partial\!\!\!/\Psi-U(\Phi,\Psi)]. \tag{1}$$

Although we suppress internal indices, Φ stands for a set of bosonic scalar fields and Ψ stands for a set of fermionic (classically anticommuting) Majorana spinor fields obeying the reality condition

$$\Psi_\alpha=C_{\alpha\beta}\bar{\Psi}_\beta \tag{2}$$

where C is a 2×2 charge conjugation matrix. A complex Dirac field may be considered as a complex combination of two Majorana fields.

In the special case when Φ and Ψ consist of one element each, and choosing $U(\Phi,\Psi)$ in the following form:

$$U(\Phi,\Psi)=\tfrac{1}{2}(V'(\Phi))^2+\tfrac{1}{2}\bar{\Psi}\Psi V''(\Phi) \tag{3}$$

where the prime on $V(\Phi)$ denotes differentiation with respect to Φ, the theory becomes invariant (Baaklini 1978a, Di Vecchia and Ferrara 1977) under the following finite supertranslations:

$$\begin{aligned}&\Phi\to\Phi+\bar{\epsilon}\Psi-\tfrac{1}{2}\bar{\epsilon}\epsilon V'(\Phi)\\&\Psi\to\Psi-(\mathrm{i}\partial\!\!\!/\Phi+V'(\Phi))\epsilon+\tfrac{1}{4}\bar{\epsilon}\epsilon(\mathrm{i}\partial\!\!\!/+V''(\Phi))\Psi.\end{aligned} \tag{4}$$

Here ϵ_α is the fermionic Majorana spinor parameter of supertranslations.

The canonical structure of the theory described by the Lagrangian (1) is very well known. However, it is instructive to recall this knowledge before expanding around finite-energy classical solutions. It should be recalled that the spinor field already presents second class constraints to be handled by Dirac's method. Moreover, a point should be brought to our attention that taking the spinor fields as anticommuting, already at the classical level, provides a 'smoother' transition to the quantum theory.

The canonical momenta conjugate to $\Phi(x,t)$ and $\Psi_\alpha(x,t)$ are respectively,

$$\mathcal{M}(x,t)=\dot{\Phi}(x,t)\qquad \bar{N}_\alpha(x,t)=(\tfrac{1}{2}\mathrm{i}\bar{\Psi}(x,t)\gamma_0)_\alpha. \tag{5}$$

The classically anticommuting field variables $\Psi_\alpha(x,t)$ and their conjugate momenta $\bar{N}_\alpha(x,t)$ are odd elements of a Grassman algebra (Berezin 1966). Consistent Poisson brackets (antibrackets) may easily be defined for them (Casalbuoni 1976, Baaklini 1977b).

We define the fundamental Poisson brackets (and antibrackets),

$$\begin{aligned}\{\Phi(x,t), \mathcal{M}(x',t)\} &= \delta(x-x') \\ \{\Psi_\alpha(x,t), \bar{N}_\beta(x',t)\} &= \delta_{\alpha\beta}\delta(x-x').\end{aligned} \tag{6}$$

The Hamiltonian is

$$H = \int \mathrm{d}x\,(\bar{N}\dot{\Psi} + \mathcal{M}\dot{\Phi}) - L = \int \mathrm{d}x\,[\tfrac{1}{2}\mathcal{M}^2 + \tfrac{1}{2}\mathrm{i}\bar{\Psi}\gamma_1\Psi' + \tfrac{1}{2}(\Phi')^2 + U(\Phi,\Psi)]. \tag{7}$$

A prime on Φ or Ψ denotes differentiation with respect to the space coordinate x.

The second equation (5) implies the weakly vanishing (≈ 0) constraints

$$\bar{K}_\alpha \equiv \bar{N}_\alpha - \tfrac{1}{2}\mathrm{i}(\bar{\Psi}\gamma_0)_\alpha \approx 0. \tag{8}$$

Using (5), we obtain

$$\{\bar{K}_\alpha(x,t), \bar{K}_\beta(y,t)\} = (\mathrm{i}C^{-1}\gamma_0)_{\alpha\beta}\delta(x-y) \equiv M_{\alpha\beta}(x-y). \tag{9}$$

Hence $\bar{K}_\alpha$ are second class constraints which can be put strongly equal to zero, thus eliminating $\bar{N}_\alpha$ from the set of basic canonical variables, by defining the one-starred Dirac brackets

$$\{f,g\}^* = \{f,g\} - \int \mathrm{d}x\,\mathrm{d}y \{f, \bar{K}_\alpha(x)\} M^{-1}_{\alpha\beta}(x-y) \{\bar{K}_\beta(y), g\} \tag{10}$$

for any two dynamical variables f and g.

Using equation (10) we obtain, for instance

$$\{\Psi_\alpha(x,t), \Psi_\beta(y,t)\}^* = (\mathrm{i}\gamma_0 C)_{\alpha\beta}\delta(x-y) \tag{11}$$

Anticommuting variables have no physical interpretation at the classical level. Their physical interpretation takes place in the quantum theory. The classical solutions of the above theory are thus restricted to $\Psi = 0$. In the latter situation, we know (Baaklini 1978a, Di Vecchia and Ferrara 1977) that by choosing $V(\Phi)$ in a specified way, in the supersymmetric theory, we would obtain the Φ^4 theory or the sine–Gordon theory in the bosonic sector. These have well known (Jackiw 1977) static finite-energy solutions, which we shall denote by $\hat{\varphi}_c(x)$.

The Hamiltonian (6) gives the energy of these solutions:

$$E = \int \mathrm{d}x\,\tfrac{1}{2}[(\hat{\varphi}_c')^2 + (V'(\hat{\varphi}_c))^2]. \tag{12}$$

From equations (4), we obtain the following supertranslated solutions parametrised by ϵ_α:

$$\begin{aligned}\varphi_c(x) &= \hat{\varphi}_c - \tfrac{1}{2}\bar{\epsilon}\epsilon V'(\hat{\varphi}_c) \\ \Psi_c(x,\epsilon) &= (\mathrm{i}\gamma_1\hat{\varphi}_c' - V'(\hat{\varphi}_c))\epsilon.\end{aligned} \tag{13}$$

The supertranslational mode is just ψ_c. It is a solution of the static fermion field equation,

$$i\gamma_1\Psi' + V''(\hat{\varphi}_c)\Psi = 0 \tag{14}$$

in the background of $\hat{\varphi}_c(x)$.

It is clear that the concept of supertranslating a classical solution is perfectly analogous to space translating it,

$$\varphi_c(x) \to \varphi_c(x+X). \tag{15}$$

In the latter case, one obtains a family of solutions parametrised by X.

3. Expansion around superparametrised classical solutions

We have previously considered (Baaklini 1978a) the construction of a quantum theory around the supertranslated solution, by expanding the field operators as

$$\begin{aligned} \Phi(x,t) &= \varphi_c(x) + \varphi(x,t) \\ \Psi(x,t) &= \psi_c(x,\epsilon) + \psi(x,t) \end{aligned} \tag{16}$$

where $\varphi_c(x)$ and $\psi_c(x,\epsilon)$ were given by (13), and $\varphi(x,t)$, $\psi(x,t)$ and $\epsilon(t)$ were taken as the basic quantum variables. We have neglected space translations as in (15) and worked to the first order in $\epsilon(t)$.

Now we deal with a more complete and general treatment. We take Φ and Ψ as multiplets of fields and the theory is not necessarily supersymmetric. We assume classical solutions $\varphi_c(x,\epsilon_\alpha^m,X_i)$ and $\psi_c(x,\epsilon_\alpha^m,X_i)$ that are parametrised by arbitrary bosonic parameters X_i, $i=1,\ldots,B$, and fermionic parameters ϵ_α^m, $m=1,\ldots,F$, the latter being Majorana spinors. We construct the quantum theory without using the explicit dependence of the classical solutions on the bosonic or the fermionic parameters.

Substituting equation (16) into equation (1), we obtain for the Lagrangian,

$$\begin{aligned} L = \int dx\bigg[\frac{1}{2}\Big(\dot{\varphi} + \frac{\partial}{\partial X_i}\varphi_c\dot{X}_i - \frac{\partial}{\partial\epsilon_\alpha^m}\varphi_c\dot{\epsilon}_\alpha^m\Big)^2 - \tfrac{1}{2}(\Phi')^2 + \tfrac{1}{2}i\bar{\Psi}\gamma_0\Big(\dot{\psi} + \frac{\partial}{\partial X_i}\psi_c\dot{X}_i + \frac{\partial}{\partial\epsilon_\alpha^m}\psi_c\dot{\epsilon}_\alpha^m\Big) \\ - \tfrac{1}{2}i\bar{\Psi}\gamma_1\Psi' - U(\Phi,\Psi)\bigg]. \end{aligned} \tag{17}$$

Thus we obtain the following canonical momenta conjugate to $\varphi(x,t)$, $X_i(t)$, $\psi_\alpha(x,t)$ and $\epsilon_\alpha^m(t)$ respectively,

$$\begin{aligned} \pi(x,t) &= \dot{\varphi} + \frac{\partial}{\partial X_i}\varphi_c\dot{X}_i - \frac{\partial}{\partial\epsilon_\alpha^m}\varphi_c\dot{\epsilon}_\alpha^m \\ P_i(t) &= \int dx\Big(\frac{\partial}{\partial X_i}\varphi_c\pi + \frac{i}{2}\bar{\Psi}\gamma_0\frac{\partial}{\partial X_i}\psi_c\Big) \\ \bar{\eta}_\alpha(x,t) &= (\tfrac{1}{2}i\bar{\Psi}\gamma_0)_\alpha \\ \bar{s}_\alpha^m(t) &= \int dx\Big(-\frac{\partial}{\partial\epsilon_\alpha^m}\varphi_c\pi + \frac{i}{2}\bar{\Psi}\gamma_0\frac{\partial}{\partial\epsilon_\alpha^m}\psi_c\Big). \end{aligned} \tag{18}$$

The Hamiltonian is

$$H=\int \mathrm{d}x(\pi\dot{\varphi}+\bar{\eta}\dot{\psi})+P_i\dot{X}_i+\bar{s}_m\dot{\epsilon}_m-L=\int \mathrm{d}x[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\Phi')^2+\tfrac{1}{2}\mathrm{i}\bar{\Psi}\gamma_1\Psi'+U(\Phi,\Psi)].$$

We define the fundamental Poisson brackets,

$$\begin{aligned}&\{\varphi(x,t),\pi(y,t)\}=\delta(x-y)\\&\{X_i(t),P_j(t)\}=\delta_{ij}\\&\{\psi_\alpha(x,t),\bar{\eta}_\beta(y,t)\}=\delta_{\alpha\beta}\delta(x-y)\\&\{\epsilon^m_\alpha(t),\bar{s}^m_\beta(t)\}=\delta_{\alpha\beta}\delta^{mn}.\end{aligned}\tag{20}$$

From equations (18), we obtain the following constraints:

$$\begin{aligned}&{}^1K_i(t)\equiv P_i(t)-\int \mathrm{d}x\left(\frac{\partial}{\partial X_i}\varphi_c\pi+\frac{\mathrm{i}}{2}\bar{\Psi}\gamma_0\frac{\partial}{\partial X_i}\psi_c\right)\approx 0\\&{}^2\bar{K}_\alpha(x,t)\equiv\bar{\eta}_\alpha(x,t)-\tfrac{1}{2}\mathrm{i}(\bar{\Psi}(x,t)\gamma_0)_\alpha\approx 0\\&{}^3\bar{K}^m_\alpha(t)\equiv\bar{s}^m_\alpha(t)+\int \mathrm{d}x\left(\frac{\partial}{\partial\epsilon^m_\alpha}\varphi_c\pi-\frac{\mathrm{i}}{2}\bar{\Psi}\gamma_0\frac{\partial}{\partial\epsilon^m_\alpha}\psi_c\right)\approx 0.\end{aligned}\tag{21}$$

The Hamiltonian (19) is defined up to the constraints (21). In order to put these constraints strongly equal to zero, we must follow Dirac's method (Dirac 1964) of introducing 'gauge-fixing' conditions for first class constraints and redefining the bracketing rules for the basic canonical variables.

4. Gauge conditions and Dirac brackets

From the third equation (20), we obtain

$$\{{}^2\bar{K}_\alpha(x,t),{}^2\bar{K}_\beta(y,t)\}=(\mathrm{i}C^{-1}\gamma_0)_{\alpha\beta}\delta(x-y).\tag{22}$$

Hence, ${}^2\bar{K}_\alpha(x,t)$ are second class constraints and can be put strongly equal to zero, thus eliminating $\bar{\eta}_\alpha(x,t)$, by defining Dirac brackets as in the previous section. In this procedure, the fundamental brackets for $\varphi(x,t)$, $\pi(x,t)$, $X_i(t)$ and $\epsilon^m_\alpha(t)$ are unaffected. The other dynamical variables will have the following one-starred Dirac brackets:

$$\begin{aligned}&\{\psi_\alpha(x,t),\psi_\beta(y,t)\}^*=(\mathrm{i}\gamma_0C)_{\alpha\beta}\delta(x-y)\\&\{\psi_\alpha(x,t),\bar{s}^m_\beta(t)\}^*=\left(-\frac{1}{2}\frac{\partial}{\partial\epsilon^m_\beta}\psi_c\right)_\alpha\\&\{\psi_\alpha(x,t),P_i(t)\}^*=\left(-\frac{1}{2}\frac{\partial}{\partial X_i}\psi_c\right)_\alpha\\&\{\bar{s}^m_\alpha(t),\bar{s}^n_\beta(t)\}^*=\int\mathrm{d}x\left(\frac{\mathrm{i}}{4}\frac{\partial}{\partial\epsilon^m_\alpha}\bar{\psi}_c\gamma_0\frac{\partial}{\partial\epsilon^n_\beta}\psi_c\right)\\&\{\bar{s}^m_\alpha(t),P_i(t)\}^*=\int \mathrm{d}x\left(\frac{\mathrm{i}}{4}\frac{\partial}{\partial\epsilon^m_\alpha}\bar{\psi}_c\gamma_0\frac{\partial}{\partial X_i}\psi_c\right)\end{aligned}\tag{23}$$

$$\{P_i, P_j\}^* = \int \mathrm{d}x\left(-\frac{\mathrm{i}}{4}\frac{\partial}{\partial X_i}\bar{\psi}_c\gamma_0\frac{\partial}{\partial X_j}\psi_c\right).$$

Using the new set of fundamental brackets, we verify the following:

$$\{{}^1K_i, {}^1K_j\} = \{{}^1K_i, {}^3\bar{K}^m_\alpha\} = \{{}^3\bar{K}^m_\alpha, {}^3\bar{K}^n_\beta\} = 0 \tag{24}$$

and

$$\{{}^1K_i, H\} = \{{}^3\bar{K}^m_\alpha, H\} = 0. \tag{25}$$

Hence, ${}^1K_i$ and ${}^3\bar{K}^m_\alpha$ are first class constraints. In order to put them strongly equal to zero, thus eliminating $P_i(t)$ and $\bar{s}^m_\alpha(t)$, we have to introduce 'gauge-fixing' conditions.

Corresponding to ${}^1K_i$, we introduce the conditions,

$${}^1C_i \equiv \int \mathrm{d}x\left(\frac{\partial}{\partial X_i}\varphi_c\varphi\right) \approx 0 \tag{26}$$

and we have

$$\{{}^1C_i, {}^1C_j\} = 0 \qquad \{{}^1K_i, {}^1C_j\} = \mu_{ij} - \Xi_{ij} \tag{27}$$

where

$$\mu_{ij} \equiv \int \mathrm{d}x\left(\frac{\partial}{\partial X_i}\varphi_c\frac{\partial}{\partial X_j}\varphi_c\right) \qquad \Xi_{ij} \equiv \int \mathrm{d}x\left(\frac{\partial}{\partial X_i}\frac{\partial}{\partial X_j}\varphi_c\cdot\varphi\right). \tag{28}$$

Hence, we can put ${}^1K_i$ as well as ${}^1C_i$ strongly equal to zero by redefining the following two-starred Dirac brackets:

$$\{f, g\}^{**} \equiv \{f, g\}^* + \{f, {}^1K_i\}^*(\mu - \Xi)^{-1}_{ij}\{{}^1C_j, g\}^* - \{f, {}^1C_i\}^*(\mu - \Xi)^{-1}_{ij}\{{}^1K_j, g\}^*. \tag{29}$$

Corresponding to ${}^3\bar{K}^m_\alpha$, we introduce the condition,

$${}^3\bar{C}^m_\alpha \equiv -\int \mathrm{d}x\left(\mathrm{i}\bar{\psi}\gamma_0\frac{\partial}{\partial\epsilon^m_\alpha}\psi_c\right) \approx 0. \tag{30}$$

Thus we have

$$\begin{aligned}&\{{}^3\bar{C}^m_\alpha(t), {}^3\bar{C}^n_\beta(t)\} = \sigma^{mn}_{\alpha\beta}\\ &\{{}^3\bar{K}^m_\alpha(t), {}^3\bar{C}^n_\beta(t)\} = \sigma^{mn}_{\alpha\beta} + \Sigma^{mn}_{\alpha\beta}\end{aligned} \tag{31}$$

where

$$\begin{aligned}\sigma^{mn}_{\alpha\beta} &\equiv \int \mathrm{d}x\left(\mathrm{i}\frac{\partial}{\partial\epsilon^m_\alpha}\bar{\psi}_c\gamma_0\frac{\partial}{\partial\epsilon^n_\beta}\psi_c\right)\\ \Sigma^{mn}_{\alpha\beta} &\equiv \int \mathrm{d}x\left(\mathrm{i}\bar{\psi}\gamma_0\frac{\partial}{\partial\epsilon^m_\alpha}\frac{\partial}{\partial\epsilon^n_\beta}\psi_c\right).\end{aligned} \tag{32}$$

Note that in obtaining equations (31), we have used the one-starred Dirac brackets of ${}^3\bar{K}^m_\alpha(t)$ and $\psi_\alpha(x, t)$, since the two-starred redefinition (29) does not modify them.

Now we can put ${}^3\bar{K}^m_\alpha(t)$ as well as ${}^3\bar{C}^m_\alpha(t)$ strongly equal to zero by redefining the three-starred Dirac brackets:

$$\begin{aligned}\{f, g\}^{***} \equiv \{f, g\}^{**} &+ \{f, {}^3\bar{K}^m_\alpha\}^{**}[(\sigma+\Sigma)^{-1}\sigma(\sigma+\Sigma)^{-1}]^{mn}_{\alpha\beta}\{{}^3\bar{K}^n_\beta, g\}^{**} \\ &- \{f, {}^3C^m_\alpha\}^{**}[(\sigma+\Sigma)^{-1}]^{mn}_{\alpha\beta}\{{}^3\bar{K}^n_\beta, g\}^{**} \\ &- \{f, {}^3\bar{K}^m_\alpha\}^{**}[(\sigma+\Sigma)^{-1}]^{mn}_{\alpha\beta}\{{}^3\bar{C}^n_\beta, g\}^{**}\end{aligned} \tag{33}$$

for any two dynamical variables f and g.

5. The quantum theory

The canonical theory is completely defined by the Hamiltonian (19) and the fundamental Dirac brackets, defined by equations (33), of the basic canonical variables $\varphi(x, t)$, $\pi(x, t)$, $X_i(t)$, $\psi_\alpha(x, t)$ and $\epsilon^m_\alpha(t)$. In the transition to the quantum theory, one considers the basic canonical variables as operators whose commutation (or anticommutation) rules are obtained from the above brackets (or antibrackets) via a factor of $(-\mathrm{i})$ times Planck's constant

The quantum state is described by a wave-functional ϕ of the canonical variables $\varphi(x, t)$, $X_i(t)$, $\psi_2(x, t)\epsilon^m_\alpha(t)$ and time

$$\phi = \phi[\varphi, \psi_\alpha, X_i, \epsilon^m_\alpha; t]. \tag{34}$$

The Hamiltonian (17) can be used to set up a Schrödinger equation

$$H\phi = \mathrm{i}\frac{\partial}{\partial t}\phi. \tag{35}$$

The constraints ${}^1K_i$, ${}^2\bar{K}_\alpha$, ${}^3\bar{K}^m_\alpha$, ${}^1C_i$ and ${}^3\bar{C}^m_\alpha$ all become strong operator conditions on the wave-functional ϕ.

In the coordinate representation, we make the following replacements, in the Hamiltonian and the constraints, for the momenta in terms of ordinary and functional derivatives:

$$\begin{aligned} &P_i(t) = \frac{1}{\mathrm{i}}\frac{\partial}{\partial X_i(t)}, \qquad &\pi(x, t) = \frac{1}{\mathrm{i}}\frac{\delta}{\delta\varphi(x, t)} \\ &\bar{s}^m_\alpha(t) = \frac{1}{\mathrm{i}}\frac{\partial}{\partial\epsilon^m_\alpha(t)}, \qquad &\bar{\eta}_\alpha(x, t) = \frac{1}{\mathrm{i}}\frac{\delta}{\delta\psi_\alpha(x, t)}. \end{aligned} \tag{36}$$

Note that ϕ is an ordinary superfield (Salam and Strathdee 1975) in $\epsilon_\alpha(t)$, in which it has a finite power expansion. It is also a functional superfield in $\psi_\alpha(x, t)$, in which it has an infinite power expansion. It represents a generalised soliton–meson state, characterised by bosonic and fermionic coordinates for the soliton as well as bosonic and fermionic mesons. Both the soliton and the meson sectors are supersymmetric when the initial theory under consideration is supersymmetric. This concludes the specification of the canonical quantum theory.

In the path integral approach to quantum theory, one has to specify (Faddeev 1970, Senjanovic 1976) the following transition amplitude:

$$R = \int \mathrm{d}p\, \mathrm{d}q \prod_i \delta(\xi_i)(\det|\{\xi_i, \xi_j\}|)^{1/2} \exp\left(\mathrm{i}\int \mathrm{d}t(p\dot{q} - H)\right) \tag{37}$$

where q and p are classical variables representing all the dynamical variables and their conjugate momenta, and ξ_i are all the constraints including the 'gauge-fixing' conditions.

Thus in our case, we obtain, after integrating over $\bar{\eta}_\alpha(x, t)$,

$$R = \int (\mathrm{d}\varphi\, \mathrm{d}\pi\, \mathrm{d}P\, \mathrm{d}X\, \mathrm{d}\psi\, \mathrm{d}\bar{s}\, \mathrm{d}\epsilon)$$

$$\times \delta\left[P_i - \int \mathrm{d}x\left(\frac{\partial}{\partial X_i}\varphi_c\pi + \frac{\mathrm{i}}{2}\bar{\Psi}\gamma_0\frac{\partial}{\partial X_i}\psi_c\right)\right]\delta\left[\int \mathrm{d}x\left(\bar{\psi}\gamma_0\frac{\partial}{\partial\epsilon_\alpha^m}\psi_c\right)\right]$$

$$\times \delta\left[\bar{s}_\alpha^m + \int \mathrm{d}x\left(\frac{\partial}{\partial\epsilon_\alpha^m}\varphi_c\pi - \frac{\mathrm{i}}{2}\bar{\Psi}\gamma_0\frac{\partial}{\partial\epsilon_\alpha^m}\psi_c\right)\right]\delta\left[\int \mathrm{d}x\left(\frac{\partial}{\partial X_i}\varphi_c\varphi\right)\right]$$

$$\times (\det|M_{ij}|\det|N_{\alpha\beta}^{mn}|)^{1/2}\exp\left\{\mathrm{i}\int \mathrm{d}t\left[\int \mathrm{d}x\left(\pi\dot{\varphi} + \frac{\mathrm{i}}{2}\bar{\Psi}\gamma_0\dot{\psi}\right) + P_i\dot{X}_i + \bar{s}\dot{\epsilon} - H\right]\right\} \tag{38}$$

where

$$M_{ij} \equiv \begin{vmatrix} 0 & (\mu - \Xi)_{ij} \\ -(\mu - \Xi)_{ij} & 0 \end{vmatrix} \qquad N_{\alpha\beta}^{mn} \equiv \begin{vmatrix} 0 & (\sigma + \Sigma)_{\alpha\beta}^{mn} \\ (\sigma + \Sigma)_{\alpha\beta}^{mn} & \sigma_{\alpha\beta}^{mn} \end{vmatrix}. \tag{39}$$

One can easily integrate over P_i and $\bar{s}_\alpha^m$ as dictated by the delta functions. The Gaussian integration over π can be done as usual. Then one is left with a functional integral in φ, X_i, ψ_α and ϵ_α^m. Feynman graph rules should be obtained for the propagators and the vertices involving the latter variables and possible ghost variables defined by the non-trivial functional measure in (38), for perturbative calculations.

6. Discussion and conclusions

We have considered in this paper the construction of a quantum theory in the background of superparametrised classical solutions with finite energy. Our formalism is general enough to cope with classical solutions containing arbitrary bosonic and fermionic parameters. However, the main such solutions of practical interest are the space-translated and supertranslated solitonic solutions. Thus classical solitonic solutions for bosonic field theories can be embedded in supersymmetric field theories. Moreover, the quantum theory can be constructed around the space-translated and the supertranslated solutions.

We have been dealing with a two-dimensional field theory, a special case of which is supersymmetric. However, the same approach and techniques can be applied to four-dimensional field theories of physical interest. For example, we can consider (Baaklini 1978b) the supersymmetric extension of spontaneously broken gauge theory (Salam and Strathdee 1975). In that theory, there are the finite-energy monopole solutions (Jackiw 1977). For the construction of the quantum theory, one should expand around the space-translated, the U(1) transformed, the gauge-transformed as well as the supertranslated monopole solutions.

One may also consider (Baaklini 1978b) finite-energy (for example, black hole) solutions in gravity and supergravity (Baaklini *et al* 1977). In this case, one should construct the quantum theory around the locally supertranslated as well as the general coordinate transformed solutions.

Acknowledgments

I would like to thank Professors J T Lewis and L O'Raifeartaigh for kind hospitality at the Dublin Institute for Advanced Studies.

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